**COM321**

**DEFINITIONS, THEOREMS, AND PROOFS**

**Definitions**

* They describe the objects and notions that we use.
* Precession is essential to any mathematical definition.
* A definition does not have to be ambiguous, that is, it has to be clear on what an object constitute of and what does not.

**Mathematical statement**

* They are made after defining objects and notions.
* They have to be precise just like the definitions.
* Like the definitions, no ambiguity about its meaning is allowed.

**Proof**

* A convincing logical argument that a statement is true.
* In mathematics, an argument must be airtight; that is, convincing in an absolute sense.

**Theorem**

* A mathematical statement proved true.
* Occasionally we prove statements that are interesting only because they assist in the proof of another, more significant statement. Such statements are called ***lemmas***.
* Occasionally a theorem or its proof may allow us to conclude easily that other, related statements are true. These statements are called ***corollaries*** of the theorem.

**FINDING PROOFS**

* Even though no one has a recipe for producing proofs, some helpful general

strategies are available.

1. Scrutinize the statement you want to prove to have a deeper understandig.
2. Try to get an intuitive, “gut” feeling of why it should be true.
3. Try to find a counterexample.

* If you find it then are done.
* If you can’t find one then it’s likely that the theorem is true.

1. Try breaking the problem into smaller easier subproblem.

**Examples: Finding Proof**

**Thorem:**

* For every graph G , the sum of degrees of all nodes in G is an even number

**Proof**

* Every edge in G contributes 1 degree to each node that it is connected to.
* Since each edge connects two nodes, each edge contributes 2 to the sum of degrees.
* If the graph has m edges, then the total sum of degrees is 2m, which is an even number.

**Theorem :**

* For any two sets A and B, A∪B =A∩B

**Proof :**

* Let X∈ a ∪ b = A ∩ B
* Then x is not in A ∪ B from the definition of the complement of a set.
* Therefore, x is not in A and x is not in B, from the definition of the union of two sets.
* In other words, x is in A and x is in B.
* Hence the definition of the intersection of two sets shows that x is in A ∩ B.

**For the other direction**;

* suppose that x is in A ∩ B. Then X is in both A and B.
* Therefore, x is not in A and x is not in B, and thus not in the union of these two sets.
* Hence x is in the complement of the union of these sets; in other words, x is in a∪b, which completes the proof of the theorem.

**TYPES OF PROOF**

***Proof by construction***

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